# Division, Rates, Place Value, Large & Small Numbers

Wrapping our minds around fractions.

Ex 25% of a quarter

↑ $\frac{1}{4}$ ↑ multiply ↑ $\frac{1}{4}$ : $\frac{1}{4} \times \frac{1}{4} = \square$

Why?

one

four quarters

one quarter

four quarters of a quarter

one quarter of a quarter (25% of a quarter)

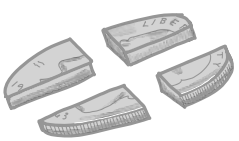
sixteen quarters of a quarter

$\frac{1}{16}$ = a quarter of a quarter = 25% of 1/4

In decimal form: 1/16 = 0.0625

A quarter of a dollar: $0.25 or 25¢ or 

- A quarter of a quarter of a dollar?



← quarter of quarter?

- $\frac{1}{16}$ dollars or _____ or _____¢

## How can we have fractions of cents?

- Fragments of pennies do not have _____
- But fractions of cents do occur in _____, and they make a difference.

Ex Gas $\$3.08^{99}$ per gallon.
Buy 18.5 gallons:

( ☐ dollars/gallon) × ( ☐ gallons) = _____ dollars

↳ Charge _____

Could round up: $\$3.09$ per gallon

( ☐ dollars/gallon) × ( ☐ gallons) = _____ dollars

↳ Charge _____

Ex Exchange Rate 1.0901 dollars per Euro

Average annual cost of BU, after aid: $\$17,597$.

How many Euros?

17,597 dollars × ☐ = _____ Euros

↳ _____ €

Note: If we rounded & used 1.09 dollars per Euro, we would get _____ €.

Ex Verizon data rate: 0.002 cents / Kilobyte
Data used: 35.893 Kilobytes
Charge should be:

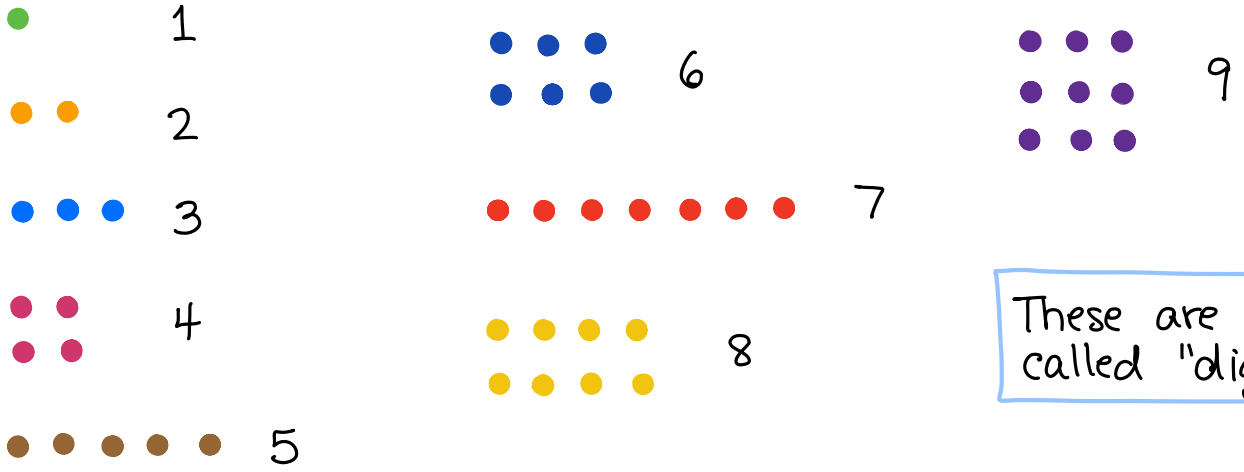
(0.002 cents/Kilobyte) × (35.893 Kilobytes)

= _____ cents → _____ or _____

George was instead charged _____.

# Place Value, Large & Small Numbers

Nice, "manageable" whole numbers are represented by a single symbol.

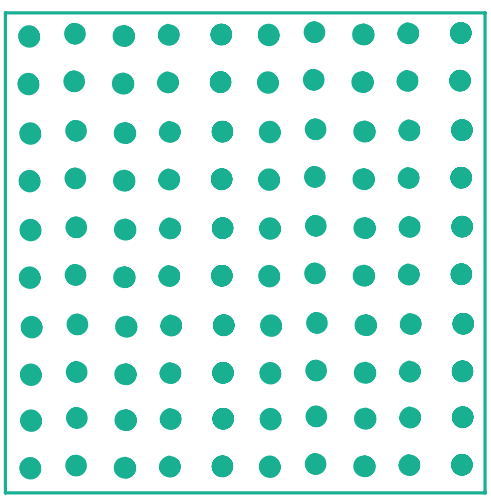



These are called "digits."

To represent larger numbers use place value; need the digit 0 for a place holder.

 one 1

 ten 10



a hundred (ten by ten square) $100 = 10^2$

and so on ... thousand 1000 = ☐

ten thousand 10000 = ☐

a hundred thousand 100 000 = ☐

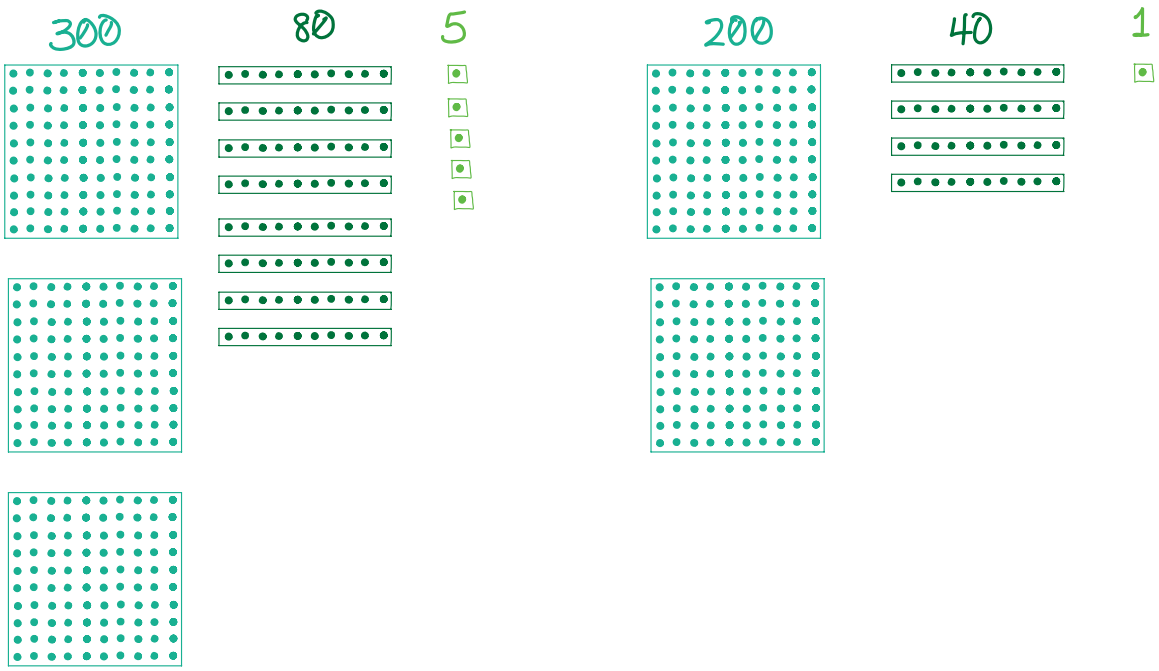
The place value system _____ & allows for _____ ways of adding & multiplying numbers.

Ex three hundred eighty-five and two hundred forty-one

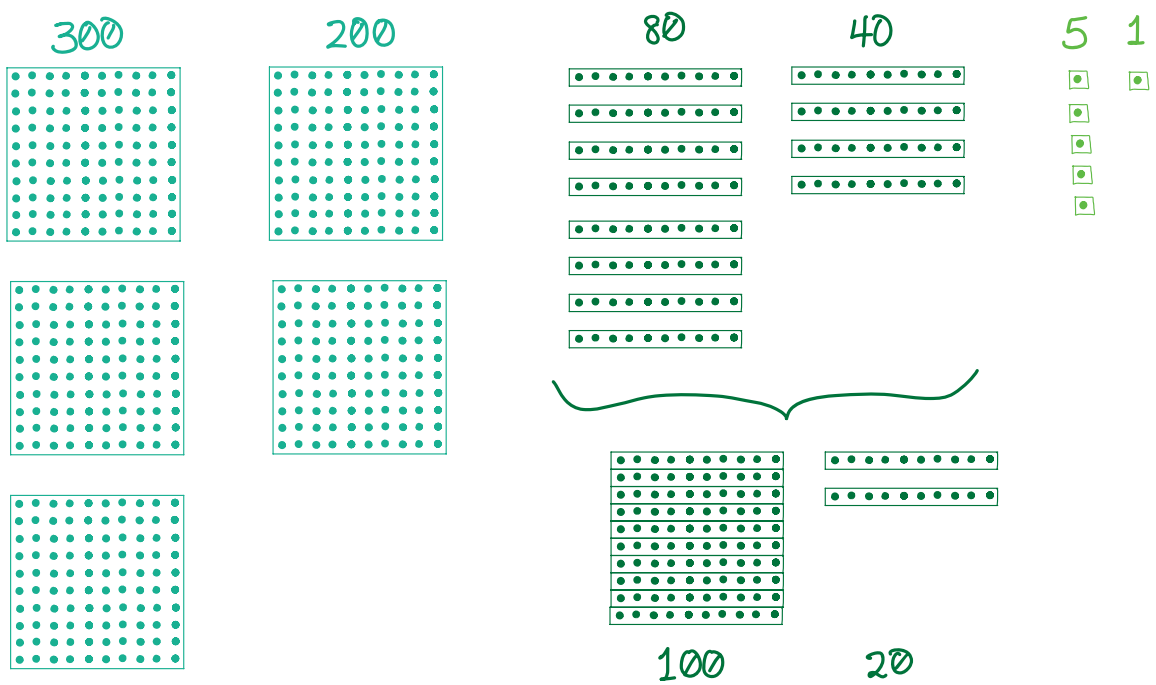
Roman : CCC L XXX V et CC XL I

300 (CCC) 50 (L) 30 (XXX) 5 (V) — 200 (CC) −10 (X before L) 50 (L) 1 (I)

Arabic : 385 + 241

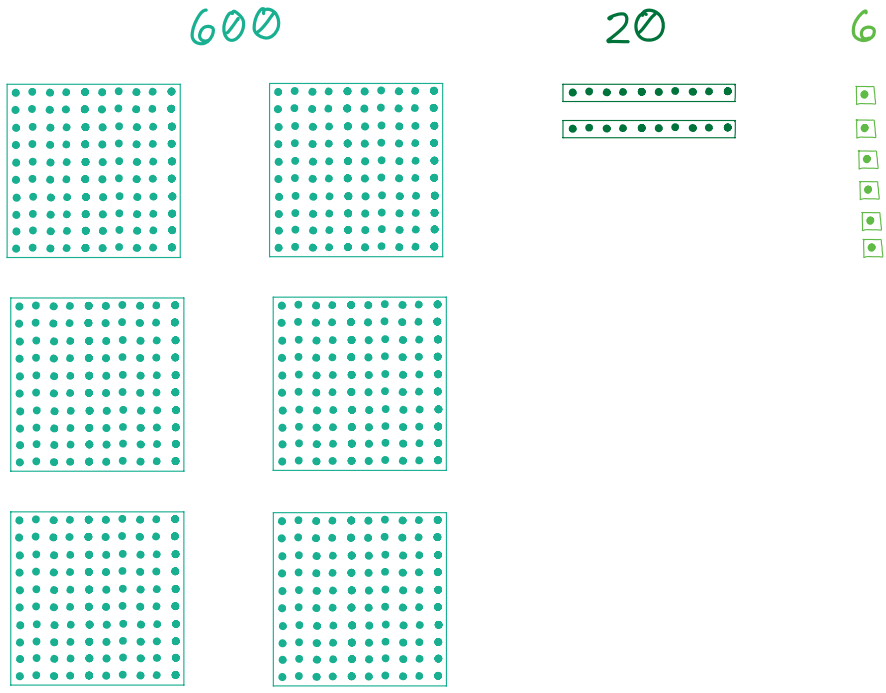



$$\begin{array}{r} 385 \\ +\ 241 \\ \hline \square\square\square \end{array}$$




$$\begin{array}{r} 385 \\ +\ 241 \\ \hline 5\ \ 6 \\ \uparrow \\ \square \end{array}$$

$$\begin{array}{r} {\scriptstyle 1\phantom{00}} \\ 385 \\ +\ 241 \\ \hline 626 \end{array}$$




The place value system provides a straightforward way to write very large numbers...

a _____ : ☐ = 1 000 000

a _____ : ☐ = 1 000 000 000

a _____ : ☐ = 1 000 000 000 000

a _____ : ☐ = 10 000 000 ... 000

... and very small numbers :

a _____ : ☐ = 0.000001

a _____ : ☐ = 0.000 000 001

⋮

Note To multiply : $10^2 \times 10^3 = 100 \times 1000 = 100\,000 =$ ____

$10^6 \times 10^8 =$ ____ = ____ (____ exponents)

To divide : $10^2/10^3 = 100/1000 = 1/10 =$ ____

$10^6/10^8 =$ ____ = ____ (____ exponents)

# Wrapping our minds around large numbers

1. Answer quickly; go with your first instinct:

   How long (in more appropriate units) is ...

   - one thousand seconds? _____
   - one million seconds? _____
   - one billion seconds? _____
   - one trillion seconds? _____

2. Now take your time and calculate:

   How long (in more appropriate units) is ...

   - one thousand seconds?

   - one million seconds?

   - one billion seconds?

   - one trillion seconds?

# Small & Large Numbers

A small portion of a large amount may be ____, ____, or ____, depending on how small the portion is & how large the amount is.

For the purposes of this discussion, let's define:

small: $< 0.01$ large: $> 1000$

(medium: in between)

Ex About 0.0052% of people in the U.S. have ALS. How many people is that?

0.0052% about 5 thousandths of one percent

= ____ about __ out of ____

U.S. population: ____

Calculate:

$(0.000052) \times (340\,000\,000) = \ldots$ (a lot of zeros :-)

OR

$(5.2 \times 10^{-5}) \times (3.4 \times 10^{8})$

$= (5.2)(3.4) \cdot 10^{\square} = \_\_ \times 10^{\square} =$ ____

About ____ people in the U.S. have ALS. (____ #)

Ex About 0.0000038% of the world's population has Barth syndrome. How many people is that?

$0.0000038\% = 3.8 \times 10^{\square}$ (about __ out of ____)

World population: 7.9 billion $= 7.9 \times 10^{\square}$

Number of people with Barth syndrome:

$(3.8 \times 10^{\square}) \times (7.9 \times 10^{\square}) = (3.8 \times 7.9) \times 10^{\square} =$ ____ = ____

About 300 people in the world have Barth syndrome. (__)

Ex Antimony in drinking water: max allowed is 0.000006 grams per liter. If the tap water in a city has the max allowable amount of antimony per liter, how much antimony will a person consume (by drinking from the tap) per year?

0.000006 grams per liter $= 6 \times 10^{\square}$ g/L

Recommended water consumption:

2.7 - 3.7 liters per day: ___ L/day (average)

Each year: (___ L/day) × (_____) = 1200 L = $1.2 \times 10^{\square}$ L

Total amount of antimony in one year:

$(6 \times 10^{\square}$ g/L) × ($1.2 \times 10^{\square}$ L) = $7.2 \times 10^{\square}$ g = _____ g

Such a person would consume about _____ grams of antimony in a year. (___ #)

## Trillions of Dollars

How much would one trillion dollars cover?

1. Estimate the cost to:

(a) build a new house for every family in Indiana

avg. house cost: _____; _____ families

(b) buy a new car for every family in Indiana

new car: _____

(c) build a new high school in every school district in Indiana

new high school: _____; school districts: ___

(d) replace every hospital in Indiana with a new one

new hospital : _____ ; ___ hospitals

2. What is the total cost of to complete all the projects in #1? Would one trillion ($1 \times 10^{12}$) dollars cover it? If so, how much money would be left over?

How large is the U.S. National Debt?

1. If the national debt was distributed equally among all the individuals in the U.S., how much debt would each person have?

U.S. National Debt : _____ , ___ $\times 10^{\square}$

U.S. population: _____ , ___ $\times 10^{\square}$

Debt per person :

2. What is the national debt as a percent of Gross Domestic Product (GDP)? (Good target : 60%)

GDP : _____ , ___ $\times 10^{\square}$

Debt as percent of GDP :

$\text{Debt}/\text{GDP} =$

U.S. National Debt is _____ of Gross Domestic Product.